On the weak solutions of a class of stochastic wave equations driven by stable space-time noises

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Outline

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2. The stochastic wave equation with Lipschitz coefficients
3. The stochastic wave equations with $\alpha-$stable noises and non–Lipschitz coefficients
4. Further discussions

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1. Some SPDEs related to super-Brownian motions
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§1 Some SPDEs related to super–BM

The superproces has ever been a quite active topic in the stochastic process studies last decades. Some survey books may refer to Dawson [1991], Perkins [2002], Li [2011],...

We begin with an interesting SPDE, which is the density field of one–dimensional super-BM. The equation is as follows

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + \sqrt{u}\dot{W}, \\ u(0) = \psi. \end{cases} \tag{1.1}$$

(Established by Konno & Shiga [1988], see also Reimers [1989]). Thus SPDEs theory provides an alternative useful tool for studying the superprocesses.

We note, the superprocesses were

(i) proposed as Markov processes (Watanabe [1968], Dawson [1975,1977], Dynkin [1988, 1989, 1990, 1991],...)

(ii) developed via Martingale Method (Roelly[1986], El-Karoui and Roelly[1991],...)

(iii) developed via SPDEs Theory (Konno and Shiga [1988], Reimers [1989], Mueller [1991, 1998],...)

Let's begin with some typical SPDEs related to superprocesses. Evans & Perkins [1994] and Dawson &Perkins [1998] proposed the following interacting SPDEs

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{1}{2}\Delta u - uv + \sqrt{u}\dot{W}^{(1)}, \\ \frac{\partial v}{\partial t} = \frac{1}{2}\Delta v - uv + \sqrt{v}\dot{W}^{(2)}, \\ u(0) = \psi_1, v(0) = \psi_2. \end{cases} \tag{1.2}$$

and

$$\begin{cases} \frac{\partial u}{\partial t} = \frac{1}{2}\Delta u + \sqrt{uv}\dot{W}^{(1)}, \\ \frac{\partial v}{\partial t} = \frac{1}{2}\Delta v + \sqrt{uv}\dot{W}^{(2)}, \\ u(0) = \psi_1, v(0) = \psi_2. \end{cases} \tag{1.3}$$

respectively.

Mytnik [2002,PTRF] proposed a SPDE with $\alpha-$stable noise:

$$\frac{\partial Y_t(x)}{\partial t} = \frac{1}{2}\Delta Y_t(x) + Y_{t-}^{\beta}(x)\dot{L}_\alpha(t,x), \quad t \geq 0, \quad x \in \mathbb{R}^d. \quad (1.4)$$

where $Y_t^{\beta}(x)$ is non–Lipschitzian and $\dot{L}_\alpha$ is not a Gaussian white noise, but $\alpha-$stable white noise.

When $\alpha\beta = 1$, $Y_t(x)$ is the density field of super–BM with stable branching mechanism.

The author proved the weak existence of the solution for (1.4) with parameters $0 < \alpha\beta < \frac{2}{d} + 1$, and $1 < \alpha < \min(2, \frac{2}{d} + 1)$.

Strum [2003, EJP] considered a broader class of SPDEs with non-Lipschitz coefficients and colored space-time noise

$$\frac{\partial u(t,x)}{\partial t} = \Delta u(t,x) + f(t,x,u(t,x)) + \sigma(t,x,u(t,x))\dot{W}(t,x), \quad t \geq 0, x \in \mathbb{R}^d, \tag{1.5}$$

where σ is non–Lipschitz, and $\dot{W}$ is white in time and is colored in space.

The author constructed a sequence of population branching systems in a random enviorment. Using a rescaling procedure and taking a limit, the author gets a weak convergence limit, which is the weak solution of (1.5). This also proved the existence of the weak solution for (1.5).

Xiong & Yang [2019,SPA] studied a class of SPDEs as follows:

$$\frac{\partial X_t(x)}{\partial t} = \frac{1}{2}\Delta X_t(x) + X_{t-}^{\beta}(x)\dot{Z}_\alpha(t,x), \quad t \geq 0, x \in \mathbb{R}^d. \quad (1.6)$$

where $\dot{Z}$ is an $\alpha-$stable colored noise without negative jumps and $\beta > 1 - \frac{1}{\alpha}$.

They proved the weak existence and pathwise uniqueness of the solution.

Outline

§2 The stochastic wave equation with Lipschitz coefficients

The wave equation is one of fundamental PDEs of hyperbolic type. The behavior of its solutions is significantly different from those of solutions to other PDEs, such as heat equations (see Dalang[2009]).

Form Newton's law of motion: $F = ma$, we might have the following relations between the acceleration and the forces on a string.

$$\kappa \frac{\partial^2 u}{\partial t^2} = F_1 - F_2 + F_3. \tag{2.1}$$

where κ is a coefficient (constant) and

$$\frac{\partial^2 u}{\partial t^2} \sim \text{acceleration}$$

$$F_1 \sim \text{elastic forces (including torsion forces)}$$

$$F_2 \sim \text{friction}$$

$$F_3 \sim \text{random impulses.}$$

Now we can mathematically write one–dimensional SPDE which is compatible with (2.1)

$$\frac{\partial^2 u(t,x)}{\partial t^2} = \frac{\partial^2 u(t,x)}{\partial x^2} - \int_0^1 k(x,y)u(t,y)dy + \dot{F}(t,x).$$

Hence, a variety of stochastic wave equations quite naturally is suggested as the model candidates in those physical problems analysis.

Here we mainly consider such stochastic wave equations, which are formally written as

$$\frac{\partial^2 u(t,x)}{\partial t^2} - \Delta u(t,x) = \beta(u) + \alpha(u)\dot{F}(t,x), \quad t \geq 0, x \in \mathbb{R}^d, \tag{2.2}$$

where $\dot{F}(t,x)$ is a space–time noise and $u(0,x) = \frac{\partial u(t,x)}{\partial t}|_{t=0} = 0$. On the other hand we might rewrite above equation in a mild form (if the latter could be well-defined)

$$\begin{aligned} u(t,x) = &\int_0^t \int_{\mathbb{R}^d} G(t-s,x-y)\beta(u(s,y))dsdy \\ &+ \int_0^t \int_{\mathbb{R}^d} G(t-s,x-y)\alpha(u(s,y))F(ds,dy), \end{aligned} \tag{2.3}$$

where $G(t,x)$ is the Green function of

$$L = \frac{\partial^2}{\partial t^2} - \Delta.$$

But, we actually have to face two issues:

(i) When $d \geq 3$, the operator $L = \frac{\partial^2}{\partial t^2} - \Delta$ does not have a function–valued Green function, (but distribution-valued). So how to define and to utilize the so–called "mild solution" ?

(ii) Even for $d = 2$, the Green function indeed exists, but the mild solution is yet not be well-defined for $\dot{F}(t, x)$ being a space–time white noise.

For these two issues, some significant solving method were suggested in the studies.

(i) Dalang [1999] constructed new stochastic integrals w.r.t martingale measures. This developed the Walsh's [1986] Martingale-measure Integrals. The key point is to use Fourier transform on distribution–valued functions (measures).

(ii) Mueller [1997], Dalang and Frangos [1998] proposed stochastic wave equations with the spatially homogeneous Gaussian noise (colored noise). They studied some properties of those SPDEs.

Some recent works on Stochastic Wave Equations include Dalang & Mueller [2009], Chen & Dalang [2015], Dalang & Humeau [2019]. Among their works, one of attractive topics is to study the intermittency phenomena on the solutions of the stochastic equations. Some of recent works on SPDEs also see Khoshnevisan et al [2017, 2018], ...

Another important way to study stochastic wave equations is based on infinite dimension analysis. We might regard the SPDEs as SDEs on an infinite dimension space (for instance, Hilbert space L^2 or Sobolev space H). Some interesting works are due to Chow [2002, 2009, 2011, 2015], Chow [2015]'s book, Brzeźniak & Maslowski [2005],...

Stimulated by those research, we ever worked on some stochastic wave equations, and here we mention two of the results obtained in our previous papers.

(1) B-T-W[2008, JDE]

We considered a damped stochastic wave equation

$$\begin{cases} u_{tt}(t,x) - \Delta u(t,x) + \kappa u_t(t,x) = \mu |u(t,x)|^\rho u(t,x) \\ \quad + g(u(t,x), u_t(t,x), Du(t,x)) \dfrac{\partial}{\partial t} W(t,x), \\ u(0,x) = u_0(x), \\ u_t(0,x) = v_0(x), \quad (t,x) \in (0,\infty) \times \mathbb{R}^d, \end{cases} \tag{2.4}$$

where $Du := (\frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_d})$ denotes the gradient of differential mapping u on $\mathbb{R}^d$.

The central objective in this paper is to study the explosive solutions of (2.4).

Theorem 1

Let $\rho > 0$ *for* $d = 1, 2$, *and* $0 < \rho \leq \frac{d}{(d-2)} - 1$ *for* $d \geq 3$. *Suppose that* $g(t, x, u) \equiv g(t, x)$ *and there exists a constant* $\alpha \in (0, \frac{\rho}{4}]$ *such that,*

(i) $(u_0, v_0)^T \in E$ *and* $\mathbf{E}\langle u_0, v_0\rangle > \frac{\kappa}{2\alpha}\mathbf{E}||u_0||_{L^2}^2 > 0$,

(ii) $||g||_{L^2(\mathbb{R}^+ \times \mathbb{R}^d)} < \infty$,

(iii) $\dfrac{\mathbf{E}||Du_0||_{L^2}^2 + c_0 Tr(Q)||g||_{L^2(\mathbb{R}^+ \times \mathbb{R}^d)}}{\mathbf{E}||u_0||_{L^{\rho+2}}^{\rho+2}} \leq \dfrac{2\mu}{\rho + 2}$.

Then for the mild solution $u(t, \cdot)$ *of (2.4) and its lifespan* ζ *in* $L^2(\mathbb{R}^d)-$*norm, either*

(1) $\mathbf{P}(\zeta < \infty) > 0$*(i.e.,* $u(t, \cdot)$ *in* $L^2(\mathbb{R}^d)-$*norm blows up in finite time with positive probability), or*

(2) there exists a positive time $\eta \in (0, t_0]$ *such that* $\lim_{t \to \eta^-} \mathbf{E}||u(t, \cdot)||_{L^2}^2 = +\infty$, *where*

$$t_0 := \frac{1}{\kappa} \log \left[\frac{2\alpha \mathbf{E}\langle u_0, v_0\rangle}{2\alpha \mathbf{E}\langle u_0, v_0\rangle - \kappa \mathbf{E}||u_0||_{L^2}^2} \right].$$

(2) B-S-W[2010, JTP]

We consider the following hyperbolic equation with a non–Gaussian Lévy noise perturbation:

$$\begin{cases} u_{tt}(t,x) + \kappa u_t(t,x) - \Delta u(t,x) = \int_{Z_1} a(u(t-,x),z)\dot{\tilde{N}}(dz,t) \\ \quad + \int_{Z\setminus Z_1} b(u(t-,x),z)\dot{N}(dz,t), \\ u(0,x) = \varphi(x), u_t(0,x) = \psi(x), \quad x \in D \\ u(t,x) = 0, \quad (t,x) \in [0,\infty) \times \partial D, \end{cases} \tag{2.5}$$

where $D \subset \mathbb{R}^d$ is a bounded open set with sufficiently regular boundary ∂D, $\kappa > 0$ denotes the damped coefficient and $Z_1 := \{z \in Z; |z| \leq 1\}$ with $Z \equiv \mathbb{R}^m$.

The random measure $\tilde{N}(dz,dt) = N(dz,dt) - \pi(dz)dt$ denotes the compensated Poisson random measure with $\pi(\{0\}) = 0$ and $\int_Z (1 \wedge |z|^2)\pi(dz) < \infty$.

Define a linear operator A by

$$Au = -\Delta u, \quad u \in D(A) = H^2(D) \cap H_0^1(D),$$

where $H^p(D)$ is the set of all functions $u \in L^2(D)$ which have generalized derivatives up to order p such that $D^\alpha u \in L^2(D)$ for all $\alpha : |\alpha \leq p|$, and $H_0^p(D)$ denotes the closure of $C_0^\infty(D)$ in $H^p(D)$. Set $H = L^2(D)$ and $V = H_0^1(D)$. Then A is a positive self–adjoint unbounded operator on H. Furthermore, we have $D(A) \subset V \subset H \subset V^*$.

Recall (2.5), it is equivalent to the system as follows:

$$\begin{cases} du(t) = v(t)dt \\ dv(t) = -[\kappa v(t) + Au(t)]dt + \int_{Z_1} a(u(t-), z)\dot{\tilde{N}}(dz, t) \\ \quad + \int_{Z \setminus Z_1} b(u(t-), z)\dot{N}(dz, t), \\ u(0) = \varphi, v(0) = \psi. \end{cases} \tag{2.6}$$

Theorem 2

Suppose that $a, b : \mathbb{R} \times Z \to \mathbb{R}$ *are measurable, and there exists a constant* $l_a > 0$ *such that*

$$a(0,z) \equiv 0,$$

$$|a(x,z) - a(y,z)|^2 \le l_a |x-y|^2 |z|^2.$$

Then for $X(0) = (\varphi, \psi) \in V \times H$, *(2.6) admits a unique strong solution* $X = \{X(t)\}_{t \ge 0} = \{(u(t), v(t))\}_{t \ge 0} \in V \times H$. *Furthermore, there exists a unique invariant measure* $\nu(\cdot)$ *on* $(V \times H, \mathcal{B}(V \times H))$ *for the transient semigroup* $\{\mathcal{P}_t\}_{t \ge 0}$.

(3) J-W-W[2020, Stochastics]

We propose a reflected parabolic SPDE driven by a fractional noise, in which the reflections occur at the point 0:

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t} = \dfrac{\partial^2 u(t,x)}{\partial x^2} + \ln(1+u(t,x)^2) \\ \qquad\qquad +\dot{B}^H(t,x) + \dot{\eta}(t,x) \quad (t,x) \in [0,T]\times[0,1] \\ u(t,0) = u(t,1) = 0, \quad t\in[0,T] \\ u(0,x) = u_0(x), \quad x \in [0,1], \end{cases} \tag{2.7}$$

where u_0 is a non–negative continuous function on $[0,1]$, $\dot{\eta}(t,x)$ is a random measure on $\mathbb{R}^\times \times [0,1]$ and $\dot{B}^H(t,x)$ is a fractional noise.

Further we also consider the following SPDEs with small perturbations.

$$\begin{cases} \dfrac{\partial u^{\epsilon}(t,x)}{\partial t} = \dfrac{\partial^2 u^{\epsilon}(t,x)}{\partial x^2} + \ln(1+u^{\epsilon}(t,x)^2) \\ \qquad\qquad +\epsilon \dot{B}^H(t,x) + \dot{\eta^{\epsilon}}(t,x) \quad (t,x) \in [0,T] \times [0,1] \\ u^{\epsilon}(t,0) = u^{\epsilon}(t,1) = 0, \quad t \in [0.T] \\ u^{\epsilon}(0,x) = u^{\epsilon}(x), \quad x \in [0,1], \end{cases} \tag{2.8}$$

Let $\mathcal{H}$ denote the Cameron–Martin space associated with the fractional white noise. Namely

$$\mathcal{H} = \left\{ h = \int_0^{\cdot} \int_0^{\cdot} \dot{h}(s,x) dsdx : \int_0^T \int_0^1 \dot{h}(s,x) dsdx < \infty \right\},$$

equipped by the norm

$$||h||_{\mathcal{H}} = \left(\int_0^T \int_0^1 \dot{h}^2(s,x) dsdx \right)^{\frac{1}{2}}.$$

For a given $h \in \mathcal{H}$, setup the following skeleton equation:

$$\begin{cases} \dfrac{\partial Z^h(t,x)}{\partial t} = \dfrac{\partial^2 Z^h(t,x)}{\partial x^2} + \ln(1 + Z^h(t,x)^2) \\ \qquad\qquad + F(\dot{h}) + \dot{\eta}^h(t,x) \quad (t,x) \in [0,T] \times [0,1] \\ Z^h(t,0) = Z^h(t,1) = 0, \quad t \in [0.T] \\ Z^h(0,x) = Z^h(x), \quad x \in [0,1], \end{cases} \tag{2.9}$$

Theorem 3

Let $H \in (\frac{1}{2}, 1)$. The reflected SPDE (2.7) admits a unique strong solution pair (u, η). Moreover the SPDE system (2.8) satisfies a large deviation principle on $C([0,T] \times [0,1])$ with the good rate function:

$$I(\psi) = \begin{cases} \inf\{\frac{1}{2}||h||^2_{\mathcal{H}};\ Z^h = \psi\}, & \textit{if } \psi \in \mathrm{Im(Z)}, \\ \infty, & \textit{otherwise.} \end{cases}$$

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§3 The stochastic wave equations with $\alpha-$stable noises and non–Lipschitz coefficients

In this section we study the non–linear stochastic wave equation

$$\begin{cases} \dfrac{\partial^2 u(t,x)}{\partial t^2} = \Delta u(t,x) + |u(t-,x)|^\beta \dot{L}_\alpha(t,x), \\ \dfrac{\partial u(t,x)}{\partial t}\Big|_{t=0} = u_0(x), \quad x \in \mathbb{R}^d, \\ u(0,x) = 0, \quad x \in \mathbb{R}^d, \end{cases} \tag{3.1}$$

where Δ be the $d-$dimensional Laplacian with index $d \in \{1,2\}$, $\beta \in (0,1)$ and $\dot{L}_\alpha$ be an $\alpha-$stable Lévy white noise with index $\alpha \in (1,2)$.

$\alpha-$stable Lévy white noise

We assume that $d \in \{1, 2\}$, and define a $\alpha-$stable Lévy random measure without big jump (a more detailed discussion may see e.g. Albeverio-Wu-Zhang [1998], Balan [2014] etc.): for $A \in \mathcal{B}([0, +\infty) \times \mathbb{R}^d)$,

$$L_\alpha(A) := \int_A \int_{\{|z| \leq 1\}} z(N(dz, dx, dt) - \nu_\alpha(dz)dxdt),$$

where N is a Poisson random measure on $\mathbb{R} \setminus \{0\} \times \mathbb{R}^d \times [0, \infty)$ with intensity measure $\nu_\alpha(dz)dxdt$, and $\nu_\alpha(dz)$ is a Lévy measure satisfying

$$\nu_\alpha(dz) = a\alpha(-z)^{-\alpha-1} 1_{(-\infty,0)}(z)dz + b\alpha z^{-\alpha-1} 1_{(0,+\infty)}(z)dz,$$

with $a + b = 1$. The $\alpha-$stable Lévy white noise $\dot{L}_\alpha$ is defined by the distributional derivative of L_α.

Define

$$M(dz, dx, dt) := N(dz, dx, dt) - \nu_\alpha(dz)dxdt,$$

then M is a compensated martingale measure and

$$L_\alpha(A) = \int_A \int_{\{|z|\le 1\}} zM(dz, dx, dt).$$

From Walsh [1986], we say that $u(t, x)$ is a mild solution of (3.1) if $u(t, x)$ satisfies

$$u(t,x) = \int_{\mathbb{R}^d} G(t, x-y)u_0(y)dy \\ + \int_0^{t+} \int_{\mathbb{R}^d} \int_{\{|z|\le 1\}} G(t-s, x-y)|u(s-, y)|^\beta zM(dz, dy, ds),$$

where $G(t, x)$ is the Green function of $L = \frac{\partial^2}{\partial t^2} - \Delta$.

We now state our main result (W.-Yan[Working paper]).

Theorem 3.1

Let $T > 0$ be fixed. Suppose that $\alpha \in (1, 2)$ and $p \in (\alpha, 2)$. Then for every $\mathcal{F}_0-$measurable $u_0 : \mathbb{R}^d \times \Omega \to [0, +\infty)$ with $\mathbf{E}[||u_0(\cdot)||_p^p] < +\infty$, there exist a weak mild solution $(u, \dot{L}_\alpha)$ of (3.1) and

$$\mathbf{E}\left[\sup_{0 \le t \le T} ||u(t, \cdot)||_p^p\right] < \infty. \tag{3.2}$$

Outline of the Proof:

(1) For $n \geq 1$, we construct a sequence of SPDEs with form as

$$\begin{cases} \dfrac{\partial^2 u_n(t,x)}{\partial t^2} = \Delta u_n(t,x) + \hat{\varphi}_n(u_n(t-,x))\dot{L}_\alpha(t,x), \\ \dfrac{\partial u_n(t,x)}{\partial t}|_{t=0} = u_0(x), \quad x \in \mathbb{R}^d, \\ u_n(0,x) = 0, \quad x \in \mathbb{R}^d. \end{cases} \tag{3.3}$$

where $\hat{\varphi}_n$ is a sequence of Lipschitz continuous functions such that

$$\lim_{n\to\infty} \hat{\varphi}_n(u) = |u|^\beta.$$

The first step is to prove that for every $\mathcal{F}_0-$measurable $u_0 : \mathbb{R}^d \times \Omega \to [0, +\infty)$ with $\mathbf{E}[||u_0(\cdot)||_p^p] < \infty$, there exists a unique mild solution (strong sense) $u_n(t,x)$ for (3.3) and

$$\sup_{0\leq t\leq T} \mathbf{E}[||u_n(t,\cdot)||_p^p] < \infty. \tag{3.4}$$

(2) We next prove the tightness of the sequence of stochastic $L^p(\mathbb{R}^d)-$valued processes $\{u^n(t,\cdot)\}_{n\geq 1}$. That is, we have to check the following tightness criterion (Ethier and Kurtz [1986])

Lemma 3.1

Let (E,ρ) be a complete and separable metric space and Let X^n be a sequence of processes with paths in $D([0,\infty),E)$ (The space of $E-$valued càdlàg processes). The sequence is tight in $D([0,\infty),E)$ if the following conditions hold:
(i) For every $\varepsilon>0$ and rational $t\geq 0$, there exists a compact set $\Gamma_\varepsilon\subset E$ such that

$$\inf_n \mathbf{P}\{X^n(t)\in\Gamma_\varepsilon\}\geq 1-\varepsilon. \tag{3.5}$$

(ii) For each $T>0$, there exists a index $p>0$ such that

$$\lim_{\delta\to 0}\sup_n \mathbf{E}\left[\sup_{0\leq t\leq T}\sup_{0\leq u\leq\delta}(\rho(X^n_{t+u},X^n_t)\wedge 1)^p\right]=0. \tag{3.6}$$

(3) Since $\{u_n(t,\cdot)\}_{n\geq 1}$ are tight in $D([0,\infty), L^p(\mathbb{R}))$, then the Prohorov's Theorem (Ethier and Kurtz [1986]) implies that $\{u_n(t,\cdot)\}_{n\geq 1}$ are relatively compact. That is, there exist a subsequence of $\{u_n(t,\cdot)\}_{n\geq 1}$, which we yet denoted by $\{u_n(t,\cdot)\}_{n\geq 1}$, converge weakly to $u(t,\cdot)$ in $D([0,\infty), L^p(\mathbb{R}^d))$.

By the Skorohod's Representation Theorem (Ethier and Kurtz [1986]), there exists a another probability space $(\tilde{\Omega}, \tilde{\mathcal{F}}, \tilde{\mathbf{P}})$ and càdlàg processes $\{\tilde{u}_n(t,\cdot)\}_{n\geq 1}$, $\tilde{u}(t,\cdot)$ and noise $\dot{\tilde{L}}_\alpha$ with intensity measure $\nu_\alpha(dz)dxdt$ defined on it with same distribution as $\{u_n(t,\cdot)\}_{n\geq 1}$, $u(t,\cdot)$ and $\dot{L}_\alpha$. Moreover, $\tilde{u}_n(t,\cdot)$ almost surely converges to $\tilde{u}(t,\cdot)$ as $n\to+\infty$.

Therefore, we can obtain a weak mild solution $(u, \dot{L}_\alpha) \overset{D}{=} (\tilde{u}, \dot{\tilde{L}}_\alpha)$.

Outline

§4 Further discussions

We proposed a class of stochastic wave equations with non-Lipschitz coefficients and with jump space-time noise, and we proved the weak existence of the solution for the equation. A subsequent question is about the uniqueness(in weak sense, or in strong sense). We are also interested in the asymptotic behaviors (states) of the solutions in infinite time. Those are our further considerations.

The end

Thank you!